

Perturbative QCD [†]

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ABSTRACT

This is the written version of a set of lectures on perturbative QCD that were delivered to a mixed audience of young theorists and experimentalists in the course of the XXII International Meeting on Fundamental Physics. These notes are virtually a verbatim transcription of the lectures. The selection of topics is somewhat arbitrary, but two basic points are emphasized: the rationale behind QCD and how ongoing experiments, such as those taking place in LEP and HERA, contribute to our understanding of strong interactions. The lectures are thus roughly divided into three parts: foundations of QCD, determination of α_s and jet physics, and deep inelastic experiments.

[†] Lectures delivered at the XXII International Meeting on Fundamental Physics “The Standard Model and Beyond”, Jaca, February 1994. To be published in the proceedings.

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1.- Introduction

It is not the purpose of these notes to give a detailed account of Quantum Chromodynamics, the theory of strong interactions. There are many good textbooks¹⁾ that are excellent at doing this job. We do not pretend to treat exhaustively any particular aspect of Quantum Chromodynamics or attempt to provide a general review of the status of this theory, either.

These lectures are essentially addressed to young experimentalists without too much theoretical background in Quantum Field Theory. We have attempted to single out amongst the theoretical foundations of QCD the ones where Quantum Chromodynamics really hinges on, trying to present them in terms as simple and physical as possible. Then we move to the connection between theory and experiment. This is not a simple problem in QCD. It is well known that the fields and particles we know how to compute with (with the simplest tool at our disposal, perturbation theory) are not those that are observed by experimentalists in their detectors due to the phenomenon of confinement. Quarks and gluons are real, but they cannot be detected as free particles as they are believed to be confined inside hadrons. In view of this is quite remarkable that there are theoretical techniques enabling us to put the theory to very stringent tests. To mention just a significative measurement, the value $\alpha_s(M_Z^2) = 0.123 \pm 0.006$ from an analysis of the process $e^+e^- \rightarrow \bar{q}q$ has been reported in this meeting²⁾. This is an impressive accuracy for a number that is not, strictly speaking, observable due to confinement. Even more so when we compare this precision to the one we had only a few years ago. Reaching this level of accuracy is one of the achievements of LEP.

Perturbative QCD can be applied to inclusive processes provided that the characteristic momentum transfer is large enough. In these lectures we will discuss in some detail these techniques and apply them to the determination of α_s through R_{had} , R_τ , or from jet topology. Another test of perturbative QCD is, of course, deep inelastic scattering. Here the commissioning of HERA has opened a new kinematical region where it will be possible to study the onset of non-perturbative effects. The exploration of this region is a fascinating subject interesting on its own right.

Due to the lack of space we have not included two sections that, in our view, should be in any general review of QCD. The first one concerns the so-called “spin of the proton” problem³⁾. Another topic that is not covered at all is the study of the photon structure functions. We direct the interested reader to ref. ⁴⁾.

We have not really made any serious attempt to provide a complete set of references. Those provided merely reflect personal tastes. A more complete bibliography can be found in some of the references, as indicated.

2.- Why QCD

Since Dirac we know that the vacuum in Quantum Field Theory is a rather peculiar object. The vacuum is defined by the property that all its negative energy states are filled, while those with positive energy are empty. A virtual photon with sufficient energy will promote a negative state to a positive one, thus creating a particle-hole (electron-positron) pair. Eventually, the positive energy state will decay filling again the hole in the Dirac sea with the emission of an off-shell photon which, in turn, will excite another pair and so on. This is illustrated in fig. 1.

Fig. 1.- The Dirac sea.

In the quantum world this process takes place virtually all the time. The uncertainty relation $\Delta E \Delta t > \hbar$ allows that for a short time a pair is created, then annihilated. The vacuum is thus constantly populated by virtual electron-positron pairs, becoming a dielectric medium at quantum scales. A medium that partially screens charges. The screening is more and more effective at larger distances since more and more virtual pairs enter the game. Therefore

$$\alpha \rightarrow \alpha_{eff}(r)$$

$$\alpha_{eff}(r) = \frac{\alpha}{1 + \frac{2}{3\pi} \alpha \log \frac{r}{r_0}}, \quad (1)$$

r_0 is some scale at which we choose to measure α . At large scales the effective electromagnetic coupling decreases, becoming larger at short distances. While it requires some work to get the detailed form of eq. 1, the reasoning behind the behaviour of the effective charge for $r \gg r_0$ is so general that it is difficult to see how it could go otherwise.

Yet, the opposite behaviour takes place in Quantum Chromodynamics. Fig. 2 shows the elastic and total cross sections for π scattering off protons. The cross sections show a rather complicate resonant behaviour at low energies but become much simpler at high energies. This suggests that strong interactions become stronger at low energies, i.e. long distances.

Fig. 2.- A typical hadronic cross-section at low energies⁵⁾

In a completely different regime deep inelastic scattering (fig. 3) showed 25 years ago the existence of hard processes inside nucleons, at very short distances. One can get a

rough picture of these processes by just assuming that quarks are approximately free if one looks at distances $\ll 1 \text{ (GeV)}^{-1}$.

Fig. 3.- The kinematics of Deep Inelastic Scattering.

One needs a rather peculiar type of theory. It must be a theory whose elementary fields are fermions (quarks), but weakly interacting at short distances. In addition we would like it to be a renormalizable theory to be able to apply the full machinery Quantum Field Theory and this pretty much forces us to work with gauge theories⁶⁾. Trivial modifications of QED will not work because we need anti-screening of charges rather than screening.

Several hints as to the way to go come from the successful quark model. We have for instance the $\Delta^{++} = |u^\uparrow u^\uparrow u^\uparrow\rangle$. Being the lightest hadron with this quark contents we expect to have the three quarks in the ground state, hence in a symmetric wave function. This is in contradiction with Fermi statistics. The contradiction can be solved if we admit the existence of a new quantum number α and

$$|u^\uparrow u^\uparrow u^\uparrow\rangle = \frac{1}{\sqrt{6}} \epsilon^{\alpha\beta\gamma} |u_\alpha^\uparrow u_\beta^\uparrow u_\gamma^\uparrow\rangle. \quad (2)$$

Indications in the same direction also come from two very different processes. One is the cross-section for $e^+e^- \rightarrow \text{hadrons}$. Assuming a free quark model (which as we mentioned should be good as the momentum transfer is large) we have

$$R_{had} = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \sim \sum_{i=1}^n Q_i^2. \quad (3)$$

The index i runs over all degrees of freedom coupling to the intermediate photon (with charge Q_i).

$$\begin{array}{lll} n=3 & u, d, s & R = \frac{2}{3} \\ n=4 & u, d, s, c & R = \frac{10}{9} \\ n=6 & u, d, s, c, b, t & R = \frac{5}{3}. \end{array} \quad (4)$$

Experimentally all these values are off by about a factor 3. The quark model can again be reconciled with experiment if we admit that the new quantum number can take 3 values and is blind to electromagnetic interactions, q^α , $\alpha = 1, 2, 3$.

Fig. 4.- The decay $\pi^0 \rightarrow \gamma\gamma$.

The other process relevant at this point is the celebrated $\pi^0 \rightarrow \gamma\gamma$ decay. This process goes dominantly through the triangle diagram shown in fig.4, with a closed loop of quarks. This diagram is also called ‘the anomaly’, for reasons that we will discuss in a while. The calculation gives⁷⁾

$$\Gamma = \frac{1}{576\pi^3} \frac{\alpha^2}{f_\pi^2} m_\pi^3 = 0.85\text{eV}. \quad (5)$$

Experimentally $\Gamma = 7.37 \pm 0.5 \text{ eV}$. The result is off by a factor $9 = 3^2$ which, again, is understood if we accept that additional degrees of freedom, invisible to both photon and pion, exist. So we learn that ordinary hadrons seem not to carry any color. Similar analysis show that color does not couple to the W or Z bosons either.

The guess is now more or less obvious. Let’s introduce a gauge symmetry acting on the new degree of freedom just as Electromagnetism acts on the electric charge with a gauge group $U(1)$. It must be a new gauge group, since color does not couple to known gauge fields. We want a group with irreducible representations of dimension 3, for the quark model to fit in. Obvious candidates are $O(3)$, $SU(2)$, $U(2)$, $U(3)$, $SU(3)$, $SO(3)$ and $Sp(2)$. Groups such as $U(2)$, $U(3)$ and $O(3)$ can be discarded right away because $\epsilon^{\alpha\beta\gamma}$ is not an invariant tensor (and we need that for the Δ^{++}). On the other hand $Sp(2) \simeq SO(3) = SU(2)/Z_2$. Neither of these have *complex* representations of dimension 3 (hence they would lead to diquark states). We are left with $SU(3)$.

3.- Lagrangian and Symmetries

The QCD lagrangian is

$$\mathcal{L}_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + i \sum_{j=1}^n \bar{\psi}_j^\alpha \gamma^\mu D_{\mu\alpha\beta} \psi_j^\beta - \sum_{j=1}^n m_j \bar{\psi}_j^\alpha \psi_{j\alpha}, \quad (6)$$

where

$$F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g f_{abc} W_\mu^b W_\nu^c, \quad (7)$$

$$D_{\mu\alpha\beta} = \delta_{\alpha\beta} \partial_\mu - ig T_{\alpha\beta}^a W_\mu^a. \quad (8)$$

The generators T^a are related to the well-known Gell-Mann matrices and act on the fundamental representation of $SU(3)$

$$T_{\alpha\beta}^a = \frac{\lambda_{\alpha\beta}^a}{2} \quad [T^a, T^b] = i f_{abc} T^c \quad (9)$$

With the structure constants f_{abc} one constructs the generators of the adjoint representation $T_{bc}^a = if_{abc}$.

\mathcal{L}_{QCD} has a local gauge invariance. If $G(x)$ is a $SU(3)$ matrix, the transformation

$$\psi(x) \rightarrow G(x)\psi(x) \quad W_\mu(x) \rightarrow G(x)W_\mu(x)G(x)^{-1} + \frac{i}{g}\partial_\mu G(x)G(x)^{-1} \quad (10)$$

leaves \mathcal{L}_{QCD} invariant. This symmetry is crucial to remove two of the four degrees of freedom in the field W_μ , ($\mu = 0, 1, 2, 3$).

To quantize the theory one must select a gauge. The bilinear part in the W_μ field is

$$\frac{1}{2}W_\mu^a(k^2 g^{\mu\nu} - k^\mu k^\nu)W_\nu^a \equiv \frac{1}{2}W_\mu^a M^{\mu\nu} W_\nu^a; \quad (11)$$

$M^{\mu\nu}$ cannot be inverted to find the propagator. The way out is to add the piece

$$\frac{-1}{2\xi}(\partial^\mu W_\mu^a)^2. \quad (12)$$

Then

$$M^{\mu\nu} = k^2 g^{\mu\nu} - (1 - \frac{1}{\xi})k^\mu k^\nu \quad (M^{-1})^{\mu\nu} = \frac{g^{\mu\nu} - (1 - \xi)\frac{k^\mu k^\nu}{k^2}}{k^2 + i\epsilon}. \quad (13)$$

We can now write Feynman diagrams. The added term eq. 12 breaks the local gauge symmetry which, generally speaking, is only recovered for S -matrix elements.

Fig. 5.- Interaction vertices in QCD. Only the color factors are shown.

\mathcal{L}_{QCD} has several types of interaction vertices. They are shown in fig. 5. Let us now consider the process $\bar{q}q \rightarrow gg$. At tree level the appropriate diagrams are shown in fig. 6.

Fig. 6.- Tree level diagrams for $\bar{q}q \rightarrow gg$.

(c) is absent in the analogous QED process $e^+e^- \rightarrow \gamma\gamma$. Due to this diagram it turns out that the above process has a bad high-energy behaviour when we sum over final state polarizations covariantly

$$P = \frac{1}{2}g_{\mu_1\nu_1}g_{\mu_2\nu_2}J^{\mu_1\mu_2}(J^{\nu_1\nu_2})^\dagger, \quad (14)$$

but it is just fine if we include transverse polarizations only

$$P = \frac{1}{2} \tau_{\mu_1 \nu_1} \tau_{\mu_2 \nu_2} J^{\mu_1 \mu_2} (J^{\nu_1 \nu_2})^\dagger. \quad (15)$$

The tensors $g_{\mu\nu}$ and $\tau_{\mu\nu}$ are obtained by summing the polarization vectors $\epsilon_\mu(\sigma)\epsilon_\nu(\sigma)$ over all polarizations or over physical ones only, respectively. If we insist in keeping a covariant formalism in which we sum over all four polarizations something must cancel this undesirable high-energy behaviour. This is accomplished by adding to \mathcal{L}_{QCD} the piece

$$-\partial^\mu \bar{\varphi}_a D_\mu^{ab} \varphi_b \quad (16)$$

with D_μ^{ab} being the same as $D_\mu^{\alpha\beta}$ in eq. 8, but with the generators of the fundamental representation of $SU(3)$ replaced by those of the adjoint one. The fields φ^a have boson-like couplings, but are defined to have Fermi statistics. They contribute with a (-1) factor to the cross-section. They are not required in abelian theories like QED, but are crucial in QCD.

Fig. 7.- Ghost contribution to $\bar{q}q \rightarrow gg$ cross-section.

If all polarizations, physical and unphysical, are summed over one must accept that φ states can be produced even if they are ghost states with unphysical statistics. The contribution from the piece we have added to the lagrangian, eq. 16, is just right to reproduce the same results we would get keeping the physical polarizations only. In practice, in internal loops we have really no choice but to keep the covariant sum over polarizations and ghosts have to be included there. It is also possible to derive the need for the introduction of ghosts from more formal arguments⁸⁾, but that would take us too far afield.

In addition to the *local* gauge symmetry we also have exact or approximate *global* symmetries in \mathcal{L}_{QCD} . The lagrangian is invariant under the global transformation

$$\psi(x) \rightarrow \exp(-i\theta I)\psi(x), \quad (17)$$

leading to baryon number conservation

$$B = \int d^3x J_0. \quad (18)$$

If all quark masses are equal there are additional symmetries and conserved currents⁹⁾

$$\psi(x) \rightarrow \exp(-i\theta^a T^a)\psi(x); \quad (19)$$

$$J_\mu = \bar{\psi} \gamma_\mu T^a \psi. \quad (20)$$

ψ now represents a column vector containing all flavours and T^a is an $SU(N_f)$ generator. Furthermore, if all quark masses vanish \mathcal{L}_{QCD} is also invariant under

$$\psi(x) \rightarrow \exp(-i\theta^a T^a \gamma_5) \psi(x). \quad (21)$$

$$J_\mu = \bar{\psi} \gamma_\mu \gamma_5 T^a \psi \quad (22)$$

are the corresponding conserved currents. In the real world quark masses are not equal, let alone zero. The two latter symmetries eqs. 19, 21 are only approximate and this only for light quarks. Therefore the hadronic world is *approximately* invariant under $U(1) \times SU(3)_V \times SU(3)_A$. $U(1)$ is always exact and $SU(3)_V$ is nothing but the vintage $SU(3)$ of Gell-Mann¹⁰⁾, which led to the quark model thirty years ago.

\mathcal{L}_{QCD} is also invariant under

$$\psi(x) \rightarrow \exp(-i\theta I \gamma_5) \psi(x) \quad (23)$$

The ‘conserved’ current is

$$J_\mu^5 = \bar{\psi} \gamma_\mu \gamma^5 \psi. \quad (24)$$

However, when one computes Green functions with insertions of the divergence of the above current, $\partial^\mu J_\mu^5$, one gets non-zero answers. The culprit is the triangle diagram, the ‘anomaly’ (the same that we met in π^0 decay, but with the two external photons replaced by gluons). In fact, a careful calculation shows that while the axial current (24) is conserved at tree level, quantum corrections spoil that conservation and, in fact,

$$\partial_\mu (\bar{\psi} \gamma^\mu \gamma^5 \psi) = \frac{g^2}{4\pi^2} \frac{N_f}{8} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu}^a F_{\alpha\beta}^a. \quad (25)$$

The key point is that there is no way of estimating the divergent momentum integral that appears in the evaluation of fig. 4 without breaking the $U(1)_A$ symmetry, and this is not a point of mathematical finesse; it has far reaching consequences. The r.h.s. of eq. 25 is itself a total divergence $\partial^\mu K_\mu$. Then the charge Q_5 verifies

$$\dot{Q}_5 = \int d^3x \partial_0 J_5^0 = \int d^3x \partial_\mu K^\mu - \int d^3x \partial_i J_5^i \quad (26)$$

In perturbation theory all fields are small perturbations from the vacuum; they decay fast enough to infinity to be able to neglect all boundary terms in the integrals. By Gauss theorem the second integral on the r.h.s. of eq. 26 drops and

$$Q_5(t = +\infty) - Q_5(t = -\infty) = \int d^4x \partial_\mu K^\mu = 0 \quad (27)$$

In a non-abelian gauge theory such as QCD there are, however, some non-perturbative gauge configurations¹¹⁾ (i.e. configurations which are not a superposition of states with a finite number of quarks and gluons) that do not possess the nice long distance behaviour that is required for eq. 27 to vanish. These make $\dot{Q}_5 \neq 0$. A conserved charge can still be

defined by the integral of $J_0^5 - K_0$, but it is not gauge invariant (the divergence of K_μ is invariant, but not K_μ itself). There is no way to have a conserved, gauge invariant axial charge in QCD. $U(1)_A$ is not a symmetry of the theory.

At this point we should probably make some contact with ongoing experiments. Can we test in some simple way that the color structure due to the non abelian theory is indeed correct? We have to take into account fig. 5 and recall some simple group relations and definitions

$$\sum_{\alpha\beta} T_{\alpha\beta}^a T_{\beta\gamma}^a = C_F \delta_{\alpha\gamma}, \quad \sum_{ac} T_{bc}^a T_{cd}^a = C_A \delta_{bd}, \quad \text{Tr}(T^a T^b) = T \delta_{ab}. \quad (28)$$

In QCD $C_F = 4/3$, $C_A = 3$ and for generators in the fundamental representation $T = T_F = 1/2$, while in the adjoint representation $T = T_A = 3$. The factors C_F, C_A, T_F and T_A appear in jet counting rules, as shown in fig. 8

Fig. 8.- Lowest order jet cross sections.

As evidenced by fig. 9 the experimental agreement between the LEP data and the QCD predictions is perfect.

Fig. 9.- QCD color factors as measured by ALEPH and DELPHI. From¹²⁾

4.- Renormalization

Beyond tree level most Feynman diagrams are ultraviolet divergent. Take for instance the first diagram of those contributing at one loop to the gluon propagator (fig. 10)

Fig. 10.- One-loop contributions to the gluon self-energy.

Neglecting external momenta, the integral over the momenta of the internal particles is of the form

$$\int \frac{d^4 k}{(2\pi)^4} \frac{k^\alpha k^\beta}{k^4} = \infty \quad (29)$$

To make sense of the theory and get a finite result we must introduce a cut-off Λ and counterterms. A typical method is to perform a subtraction at some $q^2 = -\mu^2$. For instance, for the self-energy of the gluon propagator

$$\Pi(q^2) - \Pi(-\mu^2) \equiv \Pi_R(q^2) = \text{finite}. \quad (30)$$

Fig. 11.- Adding counterterms to make Green functions finite.

Alternatively we can make sense of the integrals using dimensional regularization by continuing the dimensionality from 4 to $n = 4 + 2\epsilon$,

$$\int \frac{d^4 k}{(2\pi)^4} \rightarrow \int \frac{d^n k}{(2\pi)^n}, \quad (31)$$

and subtract just the poles in $1/\epsilon$ (minimal subtraction, MS) or also the $\gamma_E - \log 4\pi$ that always accompanies the singularity in $1/\epsilon$ (improved minimal subtraction, \overline{MS}). For instance, for the second diagram in fig. 10, namely the quark contribution to the gluon self-energy, one has the following result after computing the integral in $n = 4 + 2\epsilon$ dimensions

$$\Pi(q^2) = -\frac{\alpha_s}{6\pi} \delta_{ab} \left(\frac{1}{\epsilon} + \gamma_E + \log \frac{m^2}{4\pi\mu^2} + \dots \right). \quad (32)$$

Using the MS and \overline{MS} schemes one gets

$$\Pi_{MS}(q^2) = -\frac{\alpha_s}{6\pi} \delta_{ab} (\gamma_E + \log \frac{m^2}{4\pi\mu^2} + \dots) \quad \Pi_{\overline{MS}}(q^2) = -\frac{\alpha_s}{6\pi} \delta_{ab} (\log \frac{m^2}{\mu^2} + \dots) \quad (33)$$

Admittedly this looks totally adhoc. It would seem that we can get rid of any divergent integral by just adding the appropriate counterterms. Of course, since these are sort

of arbitrary, we would then be able to obtain any result we want. This is not so, of course. A renormalizable theory is one in which the necessary counterterms to all orders in perturbation theory are generated by redefining the fields and parametres of the original lagrangian and nothing else, and this is highly non-trivial. QCD is such a theory: if we redefine

$$\begin{aligned}
g^0 &= Z_{1YM} Z_{3YM}^{-3/2} g = \tilde{Z}_1 \tilde{Z}_3^{-1} Z_{3YM}^{-1/2} g = Z_{1F} Z_{3YM}^{-1/2} Z_{2F}^{-1} g = Z_5^{1/2} Z_{3YM}^{-1} g, \\
W_\mu^0 &= Z_{3YM}^{1/2} W_\mu, \\
\varphi^0 &= \tilde{Z}_3^{1/2} \varphi, \\
\psi^0 &= Z_{2F}^{1/2} \psi, \\
\xi_0 &= Z_6 Z_{3YM}^{-1} \xi,
\end{aligned} \tag{34}$$

it is possible by a suitable non-unique choice of the Z 's to make QCD finite. Since all Z 's are of the form $Z = 1 + \Delta Z$, with ΔZ beginning at $\mathcal{O}(g^2)$ one must replace the original \mathcal{L}_{QCD} one started with by $\mathcal{L}_{QCD} + \Delta \mathcal{L}_{QCD}$. $\Delta \mathcal{L}_{QCD}$ contains the necessary counterterms that must be added. Equivalently, we can work directly with the QCD lagrangian with all fields and parametres replaced by the 'bare' ones, defined through eq. 34. Observables will be obtained by the prescription (Λ being the ultraviolet cut-off)

$$\Gamma(p_i, g, \mu) = \lim_{\Lambda \rightarrow \infty} Z_{3YM}^{-m/2} Z_{2F}^{-k/2} \Gamma_0(p_i, g_0, \Lambda). \tag{35}$$

On the l.h.s. we have renormalized Green functions or amplitudes expressed as a function of renormalized parametres. On the r.h.s we have bare Green functions or amplitudes expressed as a function of bare parametres. In dimensional regularization

$$\Gamma(p_i, g, \mu) = \lim_{\epsilon \rightarrow 0} Z_{3YM}^{-m/2} Z_{2F}^{-k/2} \Gamma_0(p_i, g_0, \epsilon). \tag{36}$$

μ is the subtraction scale and m and k are the number of gluon and quark external lines, respectively. The effect of counterterms is to replace the dependence on the cut-off (Λ, ϵ, \dots) by a dependence on μ . Unlike in QED where the natural scale is m_e , or the Electroweak theory where the natural scale is M_W^2 , there is really no preferred way of choosing the counterterms. The only requirement is the fulfillment of the Ward identities¹³⁾. In practice MS and \overline{MS} are the most useful, particularly the latter that seems to lead to perturbative series with a faster convergence. In the MS scheme all renormalization constants are just poles in ϵ

$$Z = 1 + \frac{\alpha_s}{\pi} \frac{a}{\epsilon} + \left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{b}{\epsilon^2} + \frac{c}{\epsilon}\right) + \dots = 1 + \frac{\alpha_s^0}{\pi} \mu^{2\epsilon} \frac{a}{\epsilon} + \dots \tag{37}$$

5.- Renormalization-group Equations

Note that regulated quantities depend on a cut-off (Λ, ϵ, \dots) and that the renormalization of fields and constants through eq. 34 trades the dependence on the cut-off by some scale μ . Yet, physics cannot depend on μ at all. If you change μ you must change at the same

time the value of your renormalized parametres to make up for the difference. A simple way to encode this observation is the following. Let's write

$$\Gamma(p_i, \alpha_s, \mu) = Z_\Gamma(\mu, \epsilon) \Gamma_0(p_i, \alpha_s^0, \epsilon) \quad (38)$$

Γ_0 is obviously independent of the subtraction scale. Therefore

$$0 = \mu \frac{d}{d\mu} \Gamma_0 = Z_\Gamma^{-1} (\mu \frac{d}{d\mu} - Z_\Gamma^{-1} \mu \frac{d}{d\mu} Z_\Gamma) \Gamma. \quad (39)$$

(We neglect everywhere the dependence on the gauge parameter, as well as the quark masses. Of course they have to be properly taken into account. Physical on-shell amplitudes are ξ independent.) From eq. 39

$$(\mu \frac{\partial}{\partial \mu} + \mu \frac{d\alpha_s}{d\mu} \frac{\partial}{\partial \alpha_s} - Z_\Gamma^{-1} \mu \frac{d}{d\mu} Z_\Gamma^{-1}) \Gamma = 0, \quad (40)$$

or, defining the so-called β -function and the Green function anomalous dimension γ_Γ ,

$$\mu \frac{d\alpha_s}{d\mu} = \alpha_s \beta(\alpha_s) \quad Z_\Gamma^{-1} \mu \frac{d}{d\mu} Z_\Gamma^{-1} = \gamma_\Gamma, \quad (41)$$

$$(\mu \frac{\partial}{\partial \mu} + \beta \alpha_s \frac{\partial}{\partial \alpha_s} - \gamma_\Gamma) \Gamma(p_i, \alpha_s, \mu) = 0. \quad (42)$$

This is called the renormalization-group equation¹⁴). Let's investigate its consequences. We can always write Γ as a function of dimensionless variables by pulling out μ^D (D : dimensionality of Γ)

$$\Gamma(\lambda p_i, \alpha_s, \mu) = \mu^D F(\frac{\lambda^2 p_i p_j}{\mu^2}, \alpha_s). \quad (43)$$

Hence

$$(\lambda \frac{\partial}{\partial \lambda} + \mu \frac{\partial}{\partial \mu} - D) \Gamma(\lambda p_i, \alpha_s, \mu) = 0. \quad (44)$$

Using the renormalization-group equation we get

$$(-\frac{\partial}{\partial t} + \beta \alpha_s \frac{\partial}{\partial \alpha_s} - \gamma_\Gamma + D) \Gamma(e^t p_i, \alpha_s, \mu) = 0. \quad (45)$$

From subtraction scale independence arguments we have been able to establish an equation concerning the dependence on the external momenta. We can formally solve this equation

$$\Gamma(e^t p_i, \alpha_s(\mu), \mu) = \exp[tD - \int_0^t dt \gamma_\Gamma(\bar{\alpha}_s(t))] \times \Gamma(p_i, \bar{\alpha}_s(t), \mu), \quad (46)$$

$\bar{\alpha}_s(t)$ is just $\alpha_s(e^t \mu)$, i.e. the same coupling constant but renormalized at a different scale.

From eq. 46 we see that when we scale the external momenta in a Green function or amplitude the change is absorbed

- (i) in a multiplicative factor that depends on the anomalous dimension as well as the engineering dimension of the amplitude and
- (ii) in a redefinition of the coupling

$$\alpha_s(\mu) \rightarrow \alpha_s(e^t \mu). \quad (47)$$

The renormalization-group evolution of the parameters in the theory (in this case exemplified by the coupling constant α_s) governs the scaling behaviour. We have to find which is the evolution of α_s under a change of μ . This is actually a very simple question. We just have to solve the equation

$$\mu \frac{d\alpha_s}{d\mu} = \alpha_s \beta(\alpha_s). \quad (48)$$

Since the μ dependence of α_s is introduced via counterterms, To compute β we have just to find the relevant renormalization constants (see eq. 34).

Of course β is evaluated in perturbation theory and, accordingly, the differential equation eq. 48 is also solved in perturbation theory. The solution will only make sense as long as the expansion parameter is small. At one loop the solution of eq. 48 is

$$\alpha_s(e^t \mu) = \frac{\alpha_s(\mu)}{1 - \frac{1}{\pi} \beta_1 \alpha_s(\mu) t}. \quad (49)$$

In QCD β_1 (the first coefficient of the β - function) is

$$\beta_1 = -\frac{11}{2} + \frac{N_f}{3} \quad (50)$$

(recall that in QED $\beta_1 = 2/3$). Note that β_1 is negative if $N_f < 16$. If β_1 is negative, at larger momentum transfers, where the relevant scale will be $e^t \mu$ and not μ , α_s will actually decrease. The solution of the renormalization-group equation will actually become better and better at higher energies.

6.- Asymptotic Freedom

Whenever we speak of ‘higher’ energies we are implicitly assuming the existence of some characteristic QCD scale. Looking at eq. 48 we note that

$$t = \frac{1}{2} \log \mu^2 = \int \frac{d\alpha_s}{\alpha_s \beta(\alpha_s)} = \psi(\alpha_s) + C. \quad (51)$$

C is an integration constant. Therefore $t - \psi(\alpha_s)$ is a constant of motion along the renormalization-group trajectory. At one loop, we plug β_1 in eq. 51 and get

$$\frac{1}{2} \log \mu^2 + \frac{\pi}{\beta_1 \alpha_s(\mu)} = C \equiv \frac{1}{2} \log \Lambda_{QCD}^2 \Rightarrow \alpha_s(\mu) = \frac{-\pi}{\frac{\beta_1}{2} \log(\mu^2 / \Lambda_{QCD}^2)}. \quad (52)$$

If we renormalize at scales much larger than Λ_{QCD} the renormalized coupling constant will be small and working at one loop will be justified. Table 1 shows a number of values for α_s extracted from different LEP observables. The relevant scale here is M_Z^2 , much larger than any hadronic scale. At such energies perturbation theory is clearly meaningful.

Table 1.- Determinations of $\alpha_s(M_Z)$. From¹²⁾.

Let us recall the physical contents of eq. 46. We can write it symbolically as

Fig. 12.- Asymptotic freedom visualized.

Ignoring for a second the overall factor, when we scale up the momenta we have exactly the same amplitude, but renormalized at scale $e^t \mu$, i.e. replacing $\alpha_s(\mu)$ by $\alpha_s(e^t \mu)$. Obviously the scaled up amplitude will correspond to a theory that interacts more weakly. In the $t \rightarrow \infty$ limit we will have a free theory. This is called asymptotic freedom and it is one of the characteristic signals of strong interactions. QCD has it, while QED has not.

From fig. 12 we learn something else. Let us imagine that for some reason radiative corrections are small for some amplitude with momenta p_i and the coupling constant renormalized at scale μ . (For instance, in QED we may decide to choose the counterterms and *define* the renormalized coupling constant in such a way that the classical formula for Thompson scattering is strictly valid at all orders in perturbation theory for on-shell particles.) The renormalization group tells us is that if we scale the external momenta *and*

the renormalization scale in the way prescribed by eq. 52, radiative corrections will also be small for the scaled amplitude. Each physical process has a characteristic scale. Choosing this scale as subtraction point typically optimizes the perturbative series.

We expect the coupling constant α_s to be small at LEP energies because $M_Z \gg \Lambda_{QCD}$, but which evidence do we have that it actually runs according to eq. 49? Well, actually α_s is not small enough for two-loop effects to be completely neglected so to answer this question in a precise way we have to work a bit harder and compute the two-loop β -function. The result is

$$\beta_2 = -\frac{51}{4} + \frac{19}{12}N_f. \quad (53)$$

Incidentally, both β_1 and β_2 are scheme independent (but not β_3 and beyond). Note that, for $N_f < 8$, β_2 is also negative. Repeating the steps that led to eq. 52 we get at the two-loop level

$$\frac{1}{2} \log \mu^2 + \frac{\pi}{\beta_1 \alpha_s(\mu)} + \frac{\beta_2}{\beta_1^2} \log \alpha_s(\mu) + \mathcal{O}(\alpha_s) = \frac{1}{2} \log \Lambda_{QCD}^2, \quad (54)$$

whose solution is

$$\alpha_s(\mu) = \frac{12\pi}{(33 - 2N_f) \log(\mu^2/\Lambda_{QCD}^2)} \left[1 - 3 \frac{153 - 19N_f}{(33 - 2N_f)^2} \frac{\log \log(\mu^2/\Lambda_{QCD}^2)}{\frac{1}{2} \log(\mu^2/\Lambda_{QCD}^2)} \right] \quad (55)$$

Working in the MS or \overline{MS} scheme changes the value of $\alpha_s(\mu)$ but the difference is $\mathcal{O}(\alpha_s^2)$. Therefore the numerical value of Λ_{QCD} does actually depend on the renormalization scheme one is using (one does not see this at the one-loop level precision). For instance, at two loops Λ_{MS} and $\Lambda_{\overline{MS}}$ are related through

$$\Lambda_{MS}^2 = \frac{e^{\gamma_E}}{4\pi} \Lambda_{\overline{MS}}^2. \quad (56)$$

Eq. 55 is thus our starting point to check whether the behaviour predicted by the renormalization group has factual support. However, there is one more thing that we have to account for when we choose to work in the MS or \overline{MS} schemes. Decoupling of heavy fermions is not manifest in any of these schemes. In practice one works with the number of quarks that are excited at the energy one is. For instance, at scales well below and well above the charm threshold we have, respectively (at the one-loop level)

$$\alpha_s(\mu) = \frac{12\pi}{27 \log(\mu^2/\Lambda^2(3))} \quad \alpha_s(\mu) = \frac{12\pi}{25 \log(\mu^2/\Lambda^2(4))}. \quad (57)$$

The coupling constant is obviously continuous as we cross the threshold, but Λ_{QCD} is not. Demanding continuity on α_s leads to the matching condition

$$\Lambda(4) = \left(\frac{\Lambda(3)}{m_c} \right)^{\frac{2}{25}} \Lambda(3). \quad (58)$$

At two-loops this gets modified to¹⁵⁾

$$\Lambda(4) = \left(\frac{\Lambda(3)}{m_c} \right)^{\frac{2}{25}} \left(\log \frac{m_c^2}{\Lambda^2(3)} \right)^{-\frac{107}{1875}} \Lambda(3). \quad (59)$$

If one insists in keeping $\Lambda(3)$ beyond the charm threshold the corrections will be large; perturbation theory breaks down.

The two loop evolution of the QCD coupling constant is shown in fig. 13 for different values of $\Lambda_{\overline{MS}}$. On top, the values obtained from different experiments at different scales are shown. Asymptotic freedom is very nicely confirmed. We cannot go to energies much lower than those shown because the coupling constant grows very quickly and three-loop effects and beyond are relevant; in fact, perturbation theory becomes meaningless. In the coming sections we will discuss some of the ways of determining α_s that are shown in this plot.

Fig. 13.- Evidence for next-to-leading scaling. From¹⁶⁾

7.- Confinement

Λ_{QCD} sets a natural scale in the theory. Well above Λ_{QCD} perturbation theory makes sense. Of course perturbative QCD at large enough energies describes a world of quasi-free quarks, interacting with Coulomb-like forces. We know very well that hadronic physics is a very different world with quarks are confined into colorless hadrons. As soon as $q^2 \sim \Lambda_{QCD}^2$ perturbation theory is unreliable. It simply cannot explain confinement.

What does confinement actually mean? One popular interpretation is that ‘it is not possible to detect an isolated quark or gluon’. The problem with this definition of confinement is that in electromagnetism, which is certainly not confining, it is not possible to detect an isolated electron either. Electromagnetism (as well as Quantum Chromodynamics) is long-range (mediated by a massless particle) and plagued (just as QCD) with infrared divergences. Both QED and QCD observables have to be inclusive enough. There is one difference, of course, and that is that photons have no $U(1)$ charge, while gluons do carry $SU(3)$ charges. Far away from the interaction point you can hope to be able

to measure the electric charge carried by one particle (or rather what the experimental resolution defines as a particle which is actually the electron surrounded by a cloud of soft photons), but even if you were able to construct a detector that measured color you probably would not be able to identify in any way the color of the quark itself.

In fact, there is another definition of confinement that tells to you that the chances of actually seeing a (gluon-dressed) quark are small: ‘there is a force between quarks that does not decrease with distance’. There is indeed phenomenological evidence (which is supported by lattice analysis) that the interquark potential in QCD is of the form

$$V(r) \sim a\Lambda_{QCD}^2 r - \frac{b}{r} + \dots \quad (60)$$

The first term is a confining quark potential. The constant a has to be ~ 1 because Λ_{QCD}^2 is the only dimensional quantity at our disposal. The Coulombic part is called the Lüscher term and plays a crucial role in heavy quark spectroscopy¹⁷⁾.

We like this second definition of confinement better, because the first one is far too imprecise. In order to see that it is useful to recall a toy example suggested by Georgi¹⁸⁾. Imagine a world in which we tune Λ_{QCD} in such a way that $\alpha_s(1\text{GeV}) = 1/137$. Since the coupling constant is so small, perturbation theory works wonderfully at such energies. The proton would be a bound state of quarks (bound by Coulomb-like forces that is) with mass roughly $3m_q$. Its size would be dictated by the Bohr radius, about 1000 times the size it has in our world. The inhabitants of this world would certainly not understand the first definition of confinement, since the confinement radius would be $\sim \Lambda^{-1} \sim 10^{21}\text{cm}$. They would see quarks as you see electrons.

Even in our world the situation is somewhat similar to that of the toy world for very heavy quarks. Indeed $\alpha_s(m_t)$ is small (say ~ 0.1). The Bohr radius is $r_0 \sim 10^{-2}\text{ fm}$, much smaller than Λ_{QCD}^{-1} . The coulombic part of the interquark potential largely dominates. (At such short distances the linearly rising potential is not at work, the leading confinement effects are $\sim r^3$, as discussed by Leutwyler some time ago¹⁹⁾, but they can be safely neglected at first approximation.)

Bottom and charm are in a somewhat intermediate position. $\alpha_s(m_b)$ is still relatively small. The Bohr radius is 10^{-1} fm , smaller but comparable to Λ_{QCD}^{-1} . Spectroscopy is basically perturbative, at least for the lowest levels, but some non-perturbative effects are visible. Charm is really no-man’s land. Both perturbative and non-perturbative effects compete even for the ground state $n = 1$. For light quarks the Bohr radius is several fm and the confining potential is fully at work.

Fig. 14.- The QCD string.

The existence of a confining potential leads to very large multiplicities and jets. The situation is visualized in fig. 14. One can imagine a quark-antiquark being formed at the primary vertex then moving apart. Part of their kinetic energy is deposited in the interquark potential as they move away. Very quickly a separation r_m is reached where the energy deposited is enough to form a new quark-antiquark pair,

$$\Lambda_{QCD}^2 r_m \simeq 2m_q, \quad (61)$$

at that moment the quark-antiquark ‘string’ breaks and the process is repeated until the average relative momentum is small enough and hadronization takes place.

There is a lot of physics in the string picture. We can think of color forces being confined in some sort of tube or string joining the two moving quarks. The chromodynamic energy is thus stored in a relatively small region of space-time. If this picture is correct we should expect hadronization to take place in this region in preference to any other. This is indeed the case; in three jet events (which originate from $\bar{q}qg$, with a hard gluon) there is a clear enhancement of soft gluon and hadron production in the regions between color lines (representing the gluon by a double color line, or $\bar{q}q$ state), and a relative depletion in other regions. This phenomenon is called color coherence²⁰.

8.- R_{had}

The observable

$$R_{had} = \frac{\sigma(e^+e^- \rightarrow hadrons)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \quad (62)$$

is probably the cleanest and simplest observable in QCD. It is fully inclusive and can be computed through a dispersion relation which is symbolically depicted in fig. 15

Fig 15.- The imaginary part of the self-energy describes the $\gamma^* \rightarrow hadrons$ width.

The sum over intermediate states on the r.h.s. of fig. 15 runs over all hadronic states. We can just as well compute this sum using another resolution of the identity—the one that is provided to us by perturbative QCD, i.e. in terms of quarks and gluons. R_{had} has been computed in this way up to third order in α_s ¹

$$R_{had} = R_0 \left[1 + \frac{\alpha_s(q^2)}{\pi} + r_2 \left(\frac{\alpha_s(q^2)}{\pi} \right)^2 + r_3 \left(\frac{\alpha_s(q^2)}{\pi} \right)^3 + \dots \right], \quad (63)$$

¹ We shall write from now on $\alpha_s(\mu)$ or $\alpha_s(\mu^2)$ equivalently. We have already argued that the right renormalization scale should be the typical momentum transfer of the process.

where q^2 is the typical momentum transfer, and in the \overline{MS} scheme

$$r_2 \simeq 2.0 - 0.12N_f \quad r_3 = -6.637 - 1.200N_f - 0.005N_f^2 - 1.240 \frac{(\sum Q_i)^2}{3 \sum Q_i^2}. \quad (64)$$

For $N_f = 5$, $r_2 \simeq 1.4$ and $r_3 \simeq -12.8$. We have seen that $\alpha_s(M_Z) \simeq 0.12$. Then

$$R_{had} = R_0[1 + 0.04 + 0.002 - 0.0008 + \dots] \quad (65)$$

The convergence of the series does not look bad, but it is not very good either. The values for the coefficients that we have been just quoted correspond to massless quark; they have to be accordingly modified for heavy quarks. When this is done and comparison with experiment is done we obtain from all four LEP experiments

$$\alpha_s(M_Z) = 0.123 \pm 0.008. \quad (66)$$

The evolution of $\sigma(e^+e^- \rightarrow hadrons)$ as a function of the energy is visualized in fig. 16

Fig. 16.- $\sigma(e^+e^- \rightarrow hadrons)$. From²¹⁾

Some comments are in order here. The first one is that the convergence of the series and, ultimately, our ability to get physics out of it hinges on two points. First of all, α_s has to be small enough. A coupling constant α_s just slightly larger produces a contribution for the $\mathcal{O}(\alpha_s^3)$ term comparable to the one of $\mathcal{O}(\alpha_s^2)$, so the convergence of the series deteriorates very quickly at low energies. Secondly, the coefficients are actually scheme dependent. For instance, if we repeat the calculation but in the MS scheme we get

$$r_2^{MS} = r_2^{\overline{MS}} + (\log 4\pi - \gamma_E) \frac{33 - 2N_f}{12} \simeq 7.35 - 0.44N_f. \quad (67)$$

The convergence of the perturbative series is much worse in the MS scheme. It is not clear why it is so good in the \overline{MS} , because in Field Theory this is often the case only if one uses a physically motivated scheme (such a subtraction at some energy scale), while the reasons to use the \overline{MS} scheme are of practical order. Be as it may, this is a welcome fact.

R_{had} is an observable so it must be independent of the renormalization scheme. If we work in the MS scheme something else must change so that the net result is still the same. The thing that changes is of course the coupling constant itself

$$\alpha_s^{\overline{MS}}(\mu) - \alpha_s^{MS}(\mu) = \frac{12\pi}{(33 - 2N_f) \log(\mu^2/\Lambda_{\overline{MS}}^2)} \frac{\log 4\pi - \gamma_E}{\log(\mu^2/\Lambda_{\overline{MS}}^2)}, \quad (68)$$

which just makes up for the changes (up to log log terms), i.e.

$$1 + \frac{\alpha_s^{MS}(\mu)}{\pi} + r_2^{MS} \left(\frac{\alpha_s^{MS}(\mu)}{\pi} \right)^2 + \dots = 1 + \frac{\alpha_s^{\overline{MS}}(\mu)}{\pi} + r_2^{\overline{MS}} \left(\frac{\alpha_s^{\overline{MS}}(\mu)}{\pi} \right)^2 + \dots \quad (69)$$

This is a general feature of perturbation theory. In this framework we necessarily deal with truncated series, so independence of the subtraction point or of the scheme can only be checked up to terms of the next order in the expansion. That is (R denotes some renormalization prescription)

$$\sum_{n=0}^{\infty} c_n(R) \alpha_s(R)^n = \sum_{n=0}^{\infty} c_n(R') \alpha_s(R')^n, \quad (70)$$

but

$$\sum_{n=0}^N c_n(R) \alpha_s(R)^n \neq \sum_{n=0}^N c_n(R') \alpha_s(R')^n; \quad (71)$$

the error being of $\mathcal{O}(\alpha_s^{N+1})$.

A lot of theoretical work has been done on the issue of which is the ‘optimal’ scheme²²⁾. Some possibilities are

- (i) FAC (Fastest Apparent Convergence): choose the scheme that makes $c_N = 0$, c_N being the last computed coefficient.
- (ii) PMS (Principle of Minimal Sensitivity): demand that

$$\frac{\partial}{\partial R}(\text{observable}) = 0,$$

and work in the scheme R that fulfills this equation (which, of course, would be *any* scheme if we knew the observable exactly).

It is a fact that the quality of the series improves when one uses these methods, but unfortunately one is forced in general to use different schemes for different observables. On the basis of these analysis it has even been claimed that α_s somehow ‘freezes’ at ~ 0.3 at low energies. However, it is fair to say that the general properties of the perturbative series in QCD are so poorly understood that any method that does not directly rely on actually

computing the neglected first term in the perturbative expansion and making sure that it is small is likely to be met with skepticism. It is hard to base derivations of fundamental parametres such as α_s on ‘optimization’ techniques.

9.- R_τ

On energy considerations it is obvious that the τ is the only lepton we know heavy enough to decay into hadrons. This of course makes it a very interesting object. The inclusive decay rate $\tau \rightarrow \nu_\tau + \text{hadrons}$ can, in principle, be derived from QCD by exactly the same techniques as R_{had} . This is shown in fig. 17.

Fig. 17.- Determination of R_τ through dispersion relations.

The counterpart of R_{had} is now

$$R_\tau = \frac{\Gamma(\tau \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau \rightarrow \nu_\tau e \bar{\nu}_e)}. \quad (72)$$

At lowest order in QCD

$$R_\tau = \frac{\Gamma(\tau \rightarrow \nu_\tau d \bar{u}) + \Gamma(\tau \rightarrow \nu_\tau s \bar{u})}{\Gamma(\tau \rightarrow \nu_\tau e \bar{\nu}_e)} \simeq 3. \quad (73)$$

R_τ can be computed as a power series in α_s . Unlike in R_{had} , the typical scale will be very low (even zero) due to kinematical reasons (see fig. 17). To tackle this problem we decompose the W boson self-energy (which here plays the role the photon vacuum polarization did in R_{had}) into vector and axial parts as well as Cabibbo-allowed and Cabibbo-suppressed terms

$$\Pi^{\mu\nu} = |V_{ud}|^2(\Pi_{udV}^{\mu\nu} + \Pi_{udA}^{\mu\nu}) + |V_{us}|^2(\Pi_{usV}^{\mu\nu} + \Pi_{usA}^{\mu\nu}). \quad (74)$$

Each $\Pi^{\mu\nu}$ can be split into transverse and longitudinal parts

$$\Pi^{\mu\nu} = (-g^{\mu\nu} k^2 + k^\mu k^\nu) \Pi^{(1)} + k^\mu k^\nu \Pi^{(0)}. \quad (75)$$

For massless fermions $\Pi^{(0)} = 0$. We assume that this is the case for simplicity. For a detailed account see²³⁾. We can write

$$\begin{aligned} R_\tau &= 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \text{Im} \Pi^{(1)}(s) \\ &= 6\pi i \int_{|s|=m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \Pi^{(1)}(s) \end{aligned} \quad (76)$$

where we have used Cauchy theorem to write the integral we are interested in as a contour integral. For large $|s|$ we can compute $\Pi^{(1)}(s)$ just as we did for the hadronic vacuum polarization. One gets basically the same result (up to factors). For instance

$$\text{Im}\Pi^{(1)}(s) = \frac{1}{2\pi}(|V_{ud}|^2 + |V_{us}|^2)[1 + \frac{\alpha_s(s)}{\pi} + r_2(\frac{\alpha_s(s)}{\pi})^2 + r_3(\frac{\alpha_s(s)}{\pi})^3 + \dots]. \quad (77)$$

We also know that

$$\frac{\alpha_s(s)}{\pi} = \frac{\alpha_s(m_\tau^2)}{\pi} + \frac{\beta_1}{2} \frac{\alpha_s(m_\tau^2)}{\pi} \log \frac{s}{m_\tau^2} + \dots \quad (78)$$

So, finally

$$R_\tau = 3(|V_{ud}|^2 + |V_{us}|^2)[1 + \frac{\alpha_s(m_\tau^2)}{\pi} + (r_2 - \frac{19}{24}\beta_1)(\frac{\alpha_s(m_\tau^2)}{\pi})^2 + \dots + \text{n.p.t.}] \quad (79)$$

n.p.t. stands for non-perturbative contributions (contributions non-expressable as a power series in α_s). We did not have to worry too much about them in R_{had} because there they were characteristically suppressed by a power of the momentum transfer. Here they can be important. Fortunately, they have been found to be small²³). Anyhow, we are now in possession of an alternative way of determining α_s through τ decay. The best data comes again from LEP. A fit to experimental numbers (including mass corrections, which have been neglected throughout) gives

$$\alpha_s(m_\tau) = 0.341 \pm 0.035 \quad \Rightarrow \quad \alpha_s(M_Z) = 0.121 \pm 0.004. \quad (80)$$

Thanks to the logarithmic scaling this method provides us with the most accurate determination of α_s so far. It is also a nice way of testing next to leading scaling.

10.- Logs in QCD

Before continuing our discussion, it is convenient to make a short theoretical digression. We have seen how the scaling of α_s with the energy is logarithmic. Logs in fact play a very crucial role in Quantum Field Theory. Let us then stop and think for a second which is the origin of these logarithmic terms.

Actually there are two types of logarithms in a Field Theory such as QCD

$$\log \frac{q^2}{\mu^2} \quad \log \frac{q^2}{\lambda^2} \quad (81)$$

They have very different origin. μ is some renormalization or subtraction scale, while λ^2 can be some external momentum squared or a small (mass)² that we have given by hand to the gluon. The first type are associated to ultraviolet divergent integrals (integrals with a bad behaviour when the internal momentum is large). The second type are infrared logs and are related to Feynman diagrams with a bad behaviour when one or more external momenta vanish. Ultraviolet logs appear in any renormalizable Field Theory after renormalization.

On the contrary, infrared logs appear whenever a theory has massless particles in the spectrum (such as photons or gluons).

A given Feynman diagram can give rise to both type of singularities at the same time. This is illustrated in fig. 18

Fig. 18.- Infrared logs.

There actually two classes of infrared logs caused by massless particles. The so-called infrared divergences arise from the presence of a *soft massless* particle ($k^\mu \rightarrow 0$). For instance in the process $e^+e^- \rightarrow \mu^+\mu^-$ at the one loop level (fig. 19) we have to compute the integral

Fig. 19.- Example of diagram having an infrared divergence.

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2[(p_1+k)^2-m^2][(p_2+k)^2-m^2]}. \quad (82)$$

When $p_1^2 = p_2^2 = m^2$ the integral behaves for $k^\mu \rightarrow 0$ as

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4}. \quad (83)$$

and diverges. This divergence is unphysical so it must be cancelled by something else. The Bloch-Nordsieck theorem²⁴⁾ states that in inclusive enough cross-sections the infrared logs cancel. What do we mean by ‘inclusive enough’? A detector will not be able to discern a ‘true’ muon from a muon accompanied by a soft enough photon (with $\vec{k} \rightarrow 0$). Therefore, in addition of the diagram shown in fig. 20 we have to consider diagrams where a soft photon is radiated by the muon, square the modulus of the amplitude and integrate over the available phase space (which actually depends on the experimental cut). When this is done the result is infrared finite. The relevant diagrams are depicted in fig. 20

Fig. 20.- Real and virtual photons have to be included for IR safe results.

The other type of infrared logs are called mass singularities. They occur in theories with massless particles because two *parallel massless* particles have an invariant mass equal to zero

$$k^2 = (k_1 + k_2)^2 = \|(\omega_1 + \omega_2, 0, 0, \omega_1 + \omega_2)\| = 0. \quad (84)$$

The appearance of such a mass singularity is illustrated in fig. 21

Fig. 21.- Diagram with a mass singularity.

$$\frac{1}{(p - k)^2} = \frac{1}{p^2 + k^2 - 2k^0 p^0 + 2k^0 p^0 \cos \theta}, \quad (85)$$

the denominator vanishes when we set all particles on shell ($p^2 = k^2 = 0$) and $\theta \rightarrow 0$ (i.e. \vec{k} is parallel to \vec{p}). Even if one of the two particles is massive there is a singularity, provided the 3-momenta are parallel.

The Kinoshita-Lee-Nauenberg theorem²⁵⁾ ensures that for inclusive enough cross section the mass singularities also cancel. Both for mass singularities and for infrared divergences there is a trade-off between λ^2 , the infrared regulator of a massless particle, and the energy and angle resolution of the inclusive cross section $\Delta E, \Delta \theta$.

In practice, it is better to regulate the infrared logs using dimensional regularization (introducing λ^2 leads to difficulties with gauge invariance). Real gluon emission diagrams are regulated by performing the phase space integration in n dimensions.

There is in fact a lot of physical insight hidden in the infrared logs. We have seen that the contribution from the diagram in fig. 19 is infrared divergent, i.e. *infinite*. Yet, physical arguments tell us that the probability of finding a ‘bare’ isolated muon should be *zero*. We know this because detectors are unable to tell apart a muon from a muon plus one soft photon or indeed from a muon plus any number of soft photons. Infrared divergences in QED can be summed up and then one sees that the probability of finding an isolated

muon is indeed zero and not infinite as the one loop diagram led us to believe. Whenever a Feynman diagram is infrared divergent it means that we have forgotten something relevant.

Let us consider in QED the interaction of a charged fermion with an external source and let us expand in the number of *virtual* photons n (i.e. in the number of loops). The total amplitude will be expressed as

$$M(p, p') = \sum_{n=0}^{\infty} M_n(p, p') \quad (86)$$

then a calculation shows that

$$\begin{aligned} M_0 &= m_0, \\ M_1 &= m_0 \alpha B + m_1, \\ M_2 &= m_0 \frac{(\alpha B)^2}{2} + m_1 \alpha B + m_2, \\ &\dots \end{aligned} \quad (87)$$

The quantities m_n are IR-finite, while B is IR-divergent. The series in eq. 86 can be summed up

$$M = \exp(\alpha B) \sum_{n=0}^{\infty} m_n, \quad m_n \sim \alpha^n, \quad (88)$$

and B can be obtained just from the lowest order diagram. Introducing an IR cut-off λ , $B \sim -\log m^2/\lambda^2$, which indeed shows that when we remove the cut-off the probability of finding an isolated charged fermion is zero in QED. The addition of soft photons changes that result multiplying the total amplitude by a factor $\sim (\Delta E/\lambda)^2$. There is a trade between the infrared regulator and $\Delta E, \Delta\theta$. The latter are, of course, detector- dependent quantities.

Although only partial results exist²⁶⁾, it is believed that a similar exponentiation takes place in QCD. Due to the confinement subtleties it is unclear whether the suppression factor is compensated by radiation of soft gluons. Even if this compensation does actually take place that would not disprove confinement, only that confinement would have nothing to do with the structure of infrared singularities of the theory.

11.- Jets and α_s

The previous discussion can be summarized in the following way: due to IR singularities one is forced to consider cross sections not of individual particles in the final state, but rather of bunches of particles, each ‘hard’ quark and gluon surrounded by a ‘soft’ cloud of gluons and, perhaps, quarks. We will call these bunches ‘jets’.

The Bloch-Nordsieck and Kinoshita-Lee-Nauenberg theorems guarantee the finiteness of the cross-sections. We have to define an energy and angle resolution. For instance, if p is the momentum of a primary quark (see fig. 22) we can impose that the energy of each soft particle in its jet satisfies $k_i^0 < \epsilon p_0$ and also that $\arg(\vec{p}, \vec{k}_i) < \delta$. We will get singularities of

the form $\alpha_s \log \epsilon \log \delta$ when $\epsilon, \delta \rightarrow 0$. The specific details depend on the precise definition of the jet. Popular jet algorithms have been discussed in this Meeting²⁾.

Fig. 22.- Idealization of a jet.

The situation is unfortunately even more involved because hadrons and not quarks and gluons are detected. The evolution of the quarks and gluons produced at high momentum transfer is perturbative at first, until the average separation of the particles becomes $\mathcal{O}(\Lambda_{QCD}^{-1})$. Then the confining potential (and the string picture) takes over. Eventually one is forced for anything other than fully inclusive observables (such as R_{had}) to introduce fragmentation and hadronization models to compute the observable cross-sections. Any observable that is not fully inclusive is described by a convolution of two very different types of physics

$$\text{Observable} = \text{Perturbative} \otimes \text{Non - perturbative}$$

The perturbative part is, in principle, calculable in QCD as a power series in α_s . It is affected by some ‘theoretical’ uncertainties since most observables have been calculated up to next to leading order, and not beyond, and higher order effects can be important. In addition there is the issue of the choice of an adequate renormalization scale, which sometimes is far from obvious.

The non-perturbative part has to be modelled. Its relation to QCD and its parameters (such as α_s) is unclear. A considerable amount of cross-checking and experimental feedback is required. In general, the more inclusive the observable is the smaller the unknowns coming in from the non-perturbative part are. At LEP there are a number of observables that are widely used and where the dependence on the hadronization model is believed to be under control such as R_3 , thrust or energy-energy correlations. The obvious question is do they give consistent results? The question is partially answered in the positive in fig. 23, showing very good agreement between determinations of α_s from jet topologies.

A big improvement is obtained after the inclusion of next to leading corrections. They are clearly required; the data would be just inconsistent otherwise. Fig. 24 shows the energy evolution of R_3 , the ratio between three and two jets which is directly proportional to α_s compared to the QCD two-loop prediction. From this observable alone the precision on α_s is approximately ± 0.010 , which is, by itself, quite remarkable. Not too long ago, at PETRA energies, the determination of α_s was painstakingly difficult. Why is it so much easier at LEP energies than hitherto?

To understand this we look in some more detail at the thrust of the jets produced.

Thrust is defined as

$$T = \max \frac{|\sum_i \vec{p}_i \vec{n}_T|}{\sum_i |\vec{p}_i|} \quad (89)$$

\vec{n}_T is the thrust axis, which is varied to maximize T . T takes values between 0.5 (spherical) to $T = 1$ (complete alignment). Figure 25 shows the average thrust for a number of experiments. The LEP value is close to 0.94, higher than in any previous experiment; events are well aligned with the momentum of the primary quark. As a consequence, it is much easier to count jets at LEP than in any previous machine.

Fig. 23.- Determination of $\alpha_s(M_Z)$ from different observables by OPAL²⁷⁾. The figure includes leading and next to leading corrections and two different hadronization models. Observables are (left to right): thrust, oblateness, F-Major, C-Par, R_3 .

Fig. 24.- R_3 for different experiments.

At LEP energies we are in an energy range where perturbative QCD calculations seem

to exhibit a reasonable convergence. If we consider the average value $\alpha_s(\mu = 34 \text{ GeV}) = 0.158 \pm 0.020$ obtained from e^+e^- machines, excluding LEP (but including Tristan data), with very similar, if not identical, hadronization models, jet algorithms, etc. we see that an 30% increase in the coupling constant enlarges the experimental error by more than a factor 3. Radiative corrections rapidly become hard to control. The size of the error is crucial to be able to extract physics from the measurements of α_s . For instance, the previously quoted value of α_s leads to $\Lambda_{\overline{MS}} = 440^{+320}_{-220} \text{ MeV}$, while from the LEP results one gets $\Lambda_{\overline{MS}} = 250^{+100}_{-80} \text{ MeV}$, a much more stringent result. Incidentally, LEP is the first experiment where the $\mathcal{O}(\alpha_s^2)$ perturbative contribution is bigger than the error induced by hadronization models.

Fig. 25.- Narrowing of jets with increasing E_{cm} .

Table 2

12.- D.I.S.: Free Parton Model

A brilliant confirmation of the existence of nearly free constituents inside the nucleon was provided more than twenty years ago by a series of experiments carried out at SLAC⁽²⁸⁾. Then it became possible to scatter electrons off nucleons in fixed target experiments with a typical momentum transfer $\sim 1 - 10 \text{ (GeV)}^2$, a kinematical range unexplored until that time. The kinematics of Deep Inelastic Scattering processes is shown in fig. 3.

The virtual intermediate boson is far off its mass-shell and scatters off a quark or gluon in a time of $\mathcal{O}(1/\sqrt{-q^2})$. Typically quarks and gluons are themselves off-shell by an amount of $\mathcal{O}(\Lambda_{QCD})$. After the scattering the outgoing particles recombine into hadrons in a time of $\mathcal{O}(1/\Lambda_{QCD})$. Thus Deep Inelastic is a two-step process

- (i) Short distance scattering occurs with a large momentum transfer. Well described by perturbation theory.
- (ii) Outgoing particles recombine. Not calculable in perturbation theory. However, step (ii) can be side-stepped all together for fully inclusive rates. Then perturbation theory is adequate to describe many features of DIS.

If we place ourselves in the center of mass of the hadron and virtual intermediate boson both particles move very fast towards each other. Whatever components the hadron contains they will all have moments parallel to P^μ , up to transversal motion of $\mathcal{O}(\Lambda_{QCD})$. Let us write

$$p^\mu = xP^\mu. \quad (90)$$

The squared CM energy of the lepton and proton constituent will be

$$\hat{s} = (xP + k)^2 \simeq 2xPk \simeq xs. \quad (91)$$

We neglect masses (as well as the fact that constituents are off-shell by $\mathcal{O}(\Lambda_{QCD})$) The final momentum of the constituent is $xP + q$. Therefore

$$0 \simeq (xP + q)^2 \simeq 2xPq + q^2, \quad (92)$$

so $x = -q^2/2Pq$. If ν is the energy transfer in the LAB system, we can also write

$$x = \frac{-q^2}{2\nu m_N} \quad (93)$$

m_N being the nucleon mass. It is convenient to introduce

$$y = \frac{Pq}{Pk} = 1 - \frac{Pk'}{Pk} \quad (94)$$

In the lab frame $y = \nu/E$ and $0 \leq y \leq 1$. y is thus the relative energy loss of the colliding lepton.

Let us for the time being ignore altogether QCD interactions and let us assume that constituents of the nucleons (which we will call partons) are free. DIS will then be described by an incoherent sum over elementary processes. The partonic differential cross sections in the LAB frame will be

- $\nu q, \bar{\nu}q$ -scattering

$$\frac{d\sigma_\nu}{dy} = \left(\frac{g^2}{4\pi}\right)^2 \frac{\pi m E}{(q^2 - M_W^2)^2} [g_L^2 + g_R^2(1-y)^2], \quad (95)$$

$$\frac{d\sigma_{\bar{\nu}}}{dy} = \left(\frac{g^2}{4\pi}\right)^2 \frac{\pi m E}{(q^2 - M_W^2)^2} [g_R^2 + g_L^2(1-y)^2]. \quad (96)$$

- eq -scattering

$$\frac{d\sigma_e}{dy} = Q^2 \frac{4\pi\alpha^2 m E}{q^4} [1 + (1-y)^2]. \quad (97)$$

The neutral current sector is dominated by γ interchange below $q^2 = M_Z^2$, so we have not bothered to include Z exchange. In eq. 97 Q is the quark electric charge (in units of e) and m is the target mass. Since $p^\mu = xP^\mu$, we just take $m = xm_N$. Then, for instance,

$$\frac{d^2\sigma_e}{dxdy} = Q^2 \frac{4\pi\alpha^2 xm_N E}{q^4} [1 + (1-y)^2]. \quad (98)$$

Let $u(x)dx, d(x)dx, \dots$ be the number of u, d, \dots quarks with momentum fraction between x and $x+dx$ in a nucleon. Then $xu(x), xd(x), \dots$ will be the fraction of the nucleon momentum carried by u, d, \dots quarks. We, of course, identify quarks with partons and, since we assume that they are free, proceed to sum incoherently over the different scattering possibilities. For instance in $ep \rightarrow eX$

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha^2}{s} \frac{1 + (1-y)^2}{xy^2} \left[\frac{4}{9}(u(x) + \bar{u}(x)) + \frac{1}{9}(d(x) + \bar{d}(x)) + \frac{1}{9}(s(x) + \bar{s}(x)) \right]. \quad (99)$$

(We neglect here the possible contribution from the sea of heavy quarks in the nucleon.) Other DIS processes weigh differently quarks and antiquarks. For instance, in $\nu p \rightarrow \mu X$ if $-q^2 \ll M_W^2$ we have

$$\frac{d^2\sigma}{dxdy} = x \frac{G_F^2 s}{\pi} [c_c^2 d(x) + s_c^2 s(x) + \bar{u}(x)(1-y)^2], \quad (100)$$

with $c_c = \cos \theta_c$, $s_c = \sin \theta_c$, the cosinus and sinus of the Cabibbo angle, respectively.

The parton distribution functions $q(x)$ are quantities which at present, generally speaking, cannot be derived from QCD. The other way round is probably more interesting: we can learn a lot about the non-perturbative regime of QCD from DIS experiments through these parton distribution functions (PDF). Probably the first thing that one learns from them is that gluons are very important. From the SLAC-MIT data²⁸⁾

$$Q = U + D + S = \int_0^1 dx x (u(x) + d(x) + s(x)) \simeq 0.44, \quad (101)$$

$$\bar{Q} = \bar{U} + \bar{D} + \bar{S} = \int_0^1 dx x (\bar{u}(x) + \bar{d}(x) + \bar{s}(x)) \simeq 0.07. \quad (102)$$

The total fraction of momentum carried by quarks (and antiquarks) is only about 50%. The rest is carried by gluons, showing that although the naive quark model works very well is just a gross simplification as a model of hadrons, at least at large $-q^2$.

Another example that the quark model fails to describe some basic features of hadrons is provided by the ‘spin of the proton’ problem³⁾. μ -scattering on polarized targets shows that the fraction of the total spin of the proton that can naively be associated to constituent quarks is surprisingly small. We shall not dwell on this matter further here.

Nevertheless, there are some obvious sum rules for the parton distribution functions which can ultimately be explained in terms of the quark model. For the proton

$$\int_0^1 dx(u(x) - \bar{u}(x)) = 2, \quad (103)$$

$$\int_0^1 dx(d(x) - \bar{d}(x)) = 1, \quad (104)$$

$$\int_0^1 dx(s(x) - \bar{s}(x)) = 0. \quad (105)$$

On QCD grounds we expect that this free parton model description of the hadrons becomes more and more accurate when $-q^2 \rightarrow \infty$, $\nu \rightarrow \infty$, while keeping x fixed. This limit is known as Bjorken scaling and in the strict $-q^2 = \infty$ limit everything depends just on x .

The parton distribution functions are actually a function of the energy. At $-q^2 = \infty$ QCD is a free theory, so we can imagine quarks and gluons sharing in equal terms the total momentum, in a sort of QCD version of the equipartition theorem of Statistical Mechanics. Therefore we expect that in this limit the relation between the momentum carried by quarks and the one carried by gluons should be $N_c N_f / 2(N_c^2 - 1)$. This can be rigorously justified within QCD²⁹⁾. From the above limiting value we see and at higher energies the total momentum carried by *constituent* or *valence* quarks diminishes and that an equally important role is played by particles from the Dirac sea of the nucleon.

Let us now try to rederive the previous results in a more theoretical setting. Let us consider for instance νp scattering. Then

$$\frac{d^2\sigma}{d(-q^2)d\nu} = \frac{G_F^2 m_N}{\pi s^2} L^{\mu\nu} H_{\mu\nu}, \quad (106)$$

where

$$L^{\mu\nu} = \frac{1}{8} \text{Tr}[\gamma^\mu (1 - \gamma_5) \gamma^\alpha \gamma^\nu (1 - \gamma_5) \gamma^\beta] k_\alpha k_\beta \quad (107)$$

is the trace over the leptonic external lines, and $H_{\mu\nu}$ is given by

$$\sum_X \langle P | J_\mu(0) | X(P') \rangle \langle X(P') | J_\nu(0) | P \rangle = \int d^4z e^{iqz} \langle P | J_\mu(z) J_\nu(0) | P \rangle = \text{Im} \Pi_{\mu\nu}(q), \quad (108)$$

with

$$\Pi_{\mu\nu}(q) = \int d^4z e^{iqz} \langle P | T J_\mu(z) J_\nu(0) | P \rangle. \quad (109)$$

We decompose $H_{\mu\nu}$ as

$$H_{\mu\nu} = -g_{\mu\nu}F_1 + \frac{P_\mu P_\nu}{\nu m_N}F_2 + \frac{i}{2\nu m_N}\epsilon_{\mu\nu\rho\sigma}P^\rho q^\sigma F_3 \quad (110)$$

(If we assume that we are working with non-polarized targets P and q are the only vectors at our disposal.) F_1 , F_2 and F_3 are the nucleon structure functions. Using the kinematical relations $x = -q^2/2\nu m_N$ and $y = 2m_N\nu/s$ we get

$$d(-q^2)d\nu = \nu s dx dy, \quad (111)$$

$$\frac{d^2\sigma}{dxdy} = \frac{G_F^2 s}{2\pi} [F_1 xy^2 + F_2(1-y) - F_3 xy(1 - \frac{y}{2})]. \quad (112)$$

Let us now compare with the free parton model. We see that (restoring the νp index, to make apparent that the structure functions are process dependent)

$$\begin{aligned} F_1^{\nu p}(x) &= c_c^2(\bar{u}(x) + d(x)) + s_c^2(s(x) + \bar{u}(x)), \\ F_2^{\nu p}(x) &= 2xc_c^2(\bar{u}(x) + d(x)) + 2xs_c^2(s(x) + \bar{u}(x)), \\ F_3^{\nu p}(x) &= 2c_c^2(\bar{u}(x) - d(x)) + 2s_c^2(-s(x) + \bar{u}(x)). \end{aligned} \quad (113)$$

For other processes the actual expressions may vary but the structure functions are always linear combinations of the parton distribution functions, i.e. $F_2(x) = x \sum_i \delta_i q_i(x)$, etc. Note that in the free parton model

$$F_L(x) = F_2(x) - \frac{F_1(x)}{2x} = 0. \quad (114)$$

This is the Callan-Gross relation, which actually is not an exact one; it gets modified when the q^2 dependence is included, i.e. we depart from the strict $-q^2 = \infty$ limit. F_L in some sense measures the spin of the target. Let us assume for one second that our target has spin zero instead of one half. Then we can write with the help of just one form factor

$$\langle X(P') | J_\mu(0) | P \rangle = F(q^2)(p + p')_\mu \quad (115)$$

Plugging this back into eq. 108 we immediately observe that $F_1=0$. However, experimentally the Callan-Gross relation is well satisfied, and, on the other hand, F_2 is certainly non-zero (see fig. 26 below).

13.- Scaling Violations

Fig. 26 shows some recent data from the ZEUS collaboration at HERA³⁰⁾.

Fig. 26.- F_2 as measured by ZEUS.

It is clear from the data that there is some $Q^2 = -q^2$ dependence in the structure functions. In other words, there are violations of Bjorken scaling and actually $F_i = F_i(x, Q^2)$. The free parton model is not completely correct (no big surprise, of course). Our job is to try to understand these violations in the framework of QCD.

Let us recall that all the strong interaction effects in DIS are contained in the hadronic tensor $H_{\mu\nu}$. For $-q^2 \rightarrow \infty$ the integral in eq. 108 is dominated by the $z^2 \rightarrow 0$ behaviour, so in order to find the q^2 -dependence for large values of $-q^2$ we need to find the short distance expansion of the product of currents $J_\mu(z)J_\nu(0)$. We evaluate this expansion in three steps³¹⁾

- We decompose

$$\begin{aligned} J_\mu(z)J_\nu(0) &= (\partial_\mu\partial_\nu - g_{\mu\nu}\partial^2)O_L(z, 0) \\ &\quad + (g_{\mu\lambda}\partial_\rho\partial_\nu + g_{\rho\nu}\partial_\mu\partial_\lambda - g_{\mu\lambda}g_{\rho\nu}\partial^2 - g_{\mu\nu}\partial_\lambda\partial_\rho)O_2^{\lambda\rho}(z, 0) \\ &\quad + \dots \end{aligned} \tag{116}$$

O_2 is assumed to be symmetric in its indices. The dots stand for terms which are antisymmetric in $\mu\nu$, which we will not take into account (they would contribute to the structure function F_3 ; we will not consider it here). Current conservation is built in this expression, which is otherwise completely general.

- We expand eq. 116 in a complete basis of local operators using the Operator Product Expansion (OPE)³²⁾.

$$O_2^{\lambda\rho}(z, 0) = \sum c_{2,n}^i(z^2)z^{\mu_1} \dots z^{\mu_n} O_{2\mu_1\dots\mu_n}^{i\lambda\rho}(0), \tag{117}$$

$$O_L(z, 0) = \sum c_{L,n}^i(z^2) z^{\mu_1} \dots z^{\mu_n} O_{L\mu_1\dots\mu_n}^i(0). \quad (118)$$

There may be more than one operator with a given set of quantum numbers at a given order in the expansions eqs. 117-118 and we have included an index i . The functions c_n^i are called Wilson coefficients. Both sides of eqs. 117-118 must agree when inserted in any expectation value or Green function. On dimensional grounds, if the dimension of the current J_μ is d_0 and the one of $O_{\mu_1\dots\mu_n}^i$ is d_0^i then, as $z^2 \rightarrow 0$, the Wilson coefficients c_n^i behave as

$$c_n^i(z^2) \sim (z^2)^{-d_0 + \frac{1}{2}(d_0^i(n) - n) + 2}. \quad (119)$$

The combination $d_0^i(n) - n$ is called the twist of the operator. For $-q^2 \rightarrow \infty$ only $z^2 \rightarrow 0$ matters, hence we need to retain the operators of lowest twist, as they provide the most singular (hence dominant) behaviour. The contribution of higher twist operators is down by inverse powers of $Q^2 = -q^2$. It is convenient to define the Fourier transform of the Wilson coefficients

$$c_n^i(Q^2) \frac{2^{n+1}}{(Q^2)^{n+1}} q^{\mu_1} \dots q^{\mu_n} = \int d^4z e^{iqz} z^{\mu_1} \dots z^{\mu_n} c_n^i(z^2). \quad (120)$$

We have neglected the tensorial structures that would give less singular contributions in the $z^2 \rightarrow 0$ limit. Naively the $c_n^i(Q^2)$ are dimensionless quantities, pure numbers. In fact this is not so. The r.h.s. of eqs. 117-118 is not well defined because when we insert the local operators in a Green function we will get additional divergences. We have to renormalize them and this changes the Wilson coefficients. As usual, via the counterterms, which in perturbation theory are a power series in α_s , a logarithmic dependence in a renormalization scale μ appears. On dimensional grounds, the dependence must be through Q^2/μ^2 . Here are the scaling violations we were after.

• Finally, we need to know the expectation values of the local operators $O_{\mu_1\dots\mu_n}^i$ in the proton state

$$\langle P | O_{L\mu_1\dots\mu_n}^i(0) | P \rangle = A_{L,n}^i P_{\mu_1} \dots P_{\mu_n}, \quad (121)$$

$$\langle P | O_{2\mu_1\dots\mu_n}^{i\lambda\rho}(0) | P \rangle = A_{2,n}^i P^\lambda P^\rho P_{\mu_1} \dots P_{\mu_n}. \quad (122)$$

This is certainly not the most general decomposition of these expectation values, but other tensorial structures will eventually prove to be of higher twist. When we collect all the pieces of the calculation and put everything together, we finally get

$$F_L(n, Q^2) \equiv \int_0^1 dx x^{n-2} F_L(x, Q^2) = \sum_i A_{L,n}^i(\mu^2) c_{L,n}^i(Q^2, \mu^2), \quad (123)$$

$$F_2(n, Q^2) \equiv \int_0^1 dx x^{n-2} F_2(x, Q^2) = \sum_i A_{2,n}^i(\mu^2) c_{2,n}^i(Q^2, \mu^2). \quad (124)$$

Knowing the Wilson coefficients and the expectation values of the matrix elements we can, in the large Q^2 limit, compute the moments of the structure functions and hence the structure functions themselves. The scaling violations come about exclusively through the logarithmic dependence on Q^2/μ^2 of the Wilson coefficients.

The product $A_n(\mu^2)c_n(Q^2, \mu^2)$ is μ -independent because, as we have just seen, is an observable. As befits a renormalizable theory $c_n(Q^2, \mu^2)$ satisfies a renormalization-group type equation (we assume that there is only one operator of a given dimension and quantum numbers, so there is no mixing — we suppress the i superindex for simplicity)

$$\left(\mu \frac{\partial}{\partial \mu} + \alpha_s \beta \frac{\partial}{\partial \alpha_s} - \gamma_{O_n}\right) c_n(Q^2, \mu^2) = 0. \quad (125)$$

The quantity γ_{O_n} is the anomalous dimension of the operator. It is found by determining the combination of renormalization constants that makes it finite. Eq. 125 can be integrated out

$$c_n(Q^2, \mu^2) = c_n(\mu^2, \mu^2) \left(\frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \right)^{\frac{\gamma_{O_n}}{\beta_1}}. \quad (126)$$

Leading to following scaling behaviour for the moments of the structure functions

$$F_i(n, Q^2) = F_i(n, \mu^2) \left(\frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \right)^{\frac{\gamma_{O_n}}{\beta_1}}, \quad (127)$$

which is our final expression. Experiments agree on the whole very nicely with the scaling violations predicted by QCD. Taking into account all the subtle points of Quantum Field Theory that have gone into the analysis, this provides a beautiful empirical check of the theoretical framework

Fig. 27.- The Q^2 evolution predicted by QCD confronts experiment. From³³⁾.

14.- Altarelli-Parisi Equations

Let us now consider an alternative, and perhaps more appealing, way of deriving the evolution equation eq. 127. Let us begin by rewriting it in a differential form. Introducing the variable

$$t = \frac{1}{2} \log \frac{Q^2}{\Lambda_{QCD}^2}, \quad (128)$$

we have, for instance,

$$\frac{\partial F_2(n)}{\partial t} = -\frac{\gamma_{O_n} \alpha_s(t)}{4\pi} F_2(n). \quad (129)$$

Let us now introduce the Mellin transform of the anomalous dimensions γ_{O_n}

$$-\frac{1}{4}\gamma_{O_n} = \int_0^1 dz z^{n-1} P(z). \quad (130)$$

Using the decomposition

$$F_2(x, t) = x \sum_f \delta_f q_f(x, t) \quad (131)$$

we arrive at

$$\frac{\partial q_f(x, t)}{\partial t} = \frac{\alpha_s(t)}{\pi} \int_x^1 \frac{dy}{y} q_f(y, t) P\left(\frac{x}{y}\right) \equiv \frac{\alpha_s(t)}{\pi} q_f \otimes P. \quad (132)$$

These are the Altarelli-Parisi equations³⁴⁾. They summarize the rate of change of the parton distribution functions with t .

Let us now try to get a physical picture. We start by looking at the QCD diagrams that can contribute to this process. At leading order in α_s we have to consider the contribution from diagrams (a)-(e) in fig. 31. Since we already know that scaling violations are logarithmic we have to look for possible sources of logs. Calculations are simplest in an axial gauge. A careful evaluation shows that ultraviolet divergences are absent in this case, so they cannot provide us with the logs. Infrared divergences also cancel when all diagrams are taken into account (although each one of them is infrared divergent). There is only one mass singularity, originating from the real gluon emission diagram (d) that survives. So this is our only source of logs.

We have then to allow that the parton actually carries a different momentum fraction y and radiates a collinear gluon. Since F_2 , up to some kinematical factors, is just a cross section describable in terms of partons we can write

$$\frac{1}{x} F_2(x, t) = \sum_f \delta_f \int_x^1 dy q_f(y) w_2(x, y, t). \quad (133)$$

In the free case (no collinear gluon emitted) we simply have

$$w_2^{free} = \frac{1}{y} \delta\left(\frac{y}{x} - 1\right). \quad (134)$$

Fig. 31.- QCD diagrams contributing to DIS.

The mass singularity introduces some t dependence in w_2 . Using the expression of F_2 in terms of $q(x)$,

$$\frac{\partial q_f(x, t)}{\partial t} = \int_x^1 q_f(y) \frac{\partial w_2(x, y, t)}{\partial t} \quad (135)$$

We can iterate the process summing up an infinite ladder of diagrams (fig. 32). This is accomplished by replacing $q_f(y)$ by $q_f(y, t)$; the virginal quark is replaced by a quark that has already exchanged many collinear gluons.

Fig. 32.- ‘Handbag’ and ladder diagrams.

We have been considering F_2 , but the same procedure can be repeated for any structure function. As a simplifying hypothesis we have neglected mixing with other operators. In fact, the evolution equation is a $(2N_f + 1) \times (2N_f + 1)$ matrix. For flavour singlet operators life is more complicated; there is mixing with gluon operators and one must also consider gluon parton distribution functions as well

$$\frac{\partial q(x, t)}{\partial t} = \frac{\alpha_s(t)}{\pi} \int_x^1 \frac{dy}{y} [q(y, t) P_{q \rightarrow q}(\frac{x}{y}) + g(y, t) P_{g \rightarrow q}(\frac{x}{y})] \quad (136)$$

$$\frac{\partial g(x, t)}{\partial t} = \frac{\alpha_s(t)}{\pi} \int_x^1 \frac{dy}{y} [g(y, t) P_{g \rightarrow g}(\frac{x}{y}) + q(y, t) P_{q \rightarrow g}(\frac{x}{y})] \quad (137)$$

This is schematically depicted in fig. 33. The detailed form of the Altarelli-Parisi kernels at leading order can be found in ²⁰⁾. Next to leading expressions for them are also available in the literature.

The analysis of Deep Inelastic Scattering based either on the OPE or on the Altarelli-Parisi equations have been amongst the most clear tests of perturbative QCD and the best way of determining α_s . However, at present the value of Λ_{QCD} extracted from LEP physics is quite competitive, if not better.

Fig. 33.- Altarelli-Parisi kernels.

15.- Parton Distribution Functions

Although there are some exceptions (notably that of the pion form factor) we do not know in general how to compute the parton distribution functions, even for $-q^2 \rightarrow \infty$. Only their evolution can be reliably computed either through the Operator Product Expansion or the use of the Altarelli-Parisi equations and this for large enough values of $-q^2$.

An interesting issue is the behaviour of the parton distribution functions at the end-points $x = 0$ and $x = 1$. The large n behaviour of the moments (eq. 123-124) probes the $x \rightarrow 1$ region. Since it is natural to expect that at the kinematical boundaries the parton distribution functions vanish, one can make the following ansatz for $x \rightarrow 1$

$$q(x, Q^2) \sim A(Q^2)(1-x)^{\nu(\alpha_s(Q^2))-1} \quad (138)$$

Demanding that eq. 138 fulfills the q^2 singlet evolution equation (136) and (137) leads to

$$A(Q^2) = A_0 \frac{[\alpha_s(Q^2)]^{-d_0}}{\Gamma(1 + \nu(\alpha_s(Q^2)))} \quad \nu(\alpha_s) = \nu_0 - \frac{16}{33 - 2N_f} \log \alpha_s(Q^2), \quad (139)$$

$$d_0 = \frac{16}{33 - 2N_f} \left(\frac{3}{4} - \gamma_E \right) \quad (140)$$

Likewise, for the gluons we have

$$g(x, Q^2) \sim A'_0 \frac{[\alpha_s(Q^2)]^{-d_0}}{\Gamma(2 + \nu(\alpha_s(Q^2)))} \frac{(1-x)^{\nu(\alpha_s(Q^2))}}{\log(1-x)} \quad (141)$$

The constants A_0, A'_0 and ν_0 are not calculable on perturbative QCD and depend on the specific operator. d_0 is universal.

When $x \rightarrow 1$ the gluon distribution functions approach zero more rapidly than the quark ones. For large values of x the quark contents of nucleons is the relevant one. For small values of x the opposite behaviour takes place, the gluon distribution function

eventually becomes dominant. At LHC the cross-section will be greatly dominated by low- x physics and the important process there will be gluon-gluon scattering. At the Tevatron the quark contents of protons and antiprotons is still dominant.

To get a semi-quantitative (but not totally satisfactory) picture of the low- x behaviour of parton distribution functions, one inverts eqs. 123-124 via an inverse Mellin transform. Then

$$F_2(x, t) = \frac{1}{2\pi i} \int dn x^{-(n-1)} F_2(n, t) \quad (142)$$

The evolution of $F_2(n, t)$ is known. We can write

$$F_2(n, t) = \exp[f(n, t)] F_2(n, t_0) \quad (143)$$

Then we proceed to evaluate the r.h.s of eq. 142 by the saddle point method. Of course this requires that $\log(1/x)$ is large and this is the reason why this procedure gives only the small x behaviour. The solution is parametrized in terms of $F_2(n_0, t_0)$, n_0 being the solution of the saddle point equation for n . Working things out one finds that indeed the gluon parton distribution function is dominant for low x behaving as

$$g(x) \sim \frac{1}{x} \exp \sqrt{C(Q^2) \log \frac{1}{x}} \quad (144)$$

where $C(Q^2)$ is calculable. Unfortunately, this answer is not totally satisfactory because something must stop the growth in $g(x)$ for low x , or else one runs into unitarity problems sooner or later, and thus eq. 144 it is not credible all the way to $x = 0$. Technically speaking, there must be corrections that destabilize the saddle point solution. Physically, the uncontrolled growth of the gluon distribution is an infrared instability. The density of soft gluons is too large. Shadowing and non-linear evolution equations are the buzzwords here³⁵⁾.

Low x means $-q^2$ large, but fixed, and $\nu \rightarrow \infty$. This is the Regge limit and we understand very little of QCD in this regime. There are several parametrizations³⁶⁾ of the quark and gluon distribution functions that fit the experimental results rather well ($\sim 1\%$) in the range $1 > x > 10^{-2}$. These parametrizations are obtained in the following way. One fits the experimental data at some relatively low value of Q^2 using, for instance, a form for the parton distribution function inspired in the quark model, say $q(x) = Ax^a(1-x)^b$. Then one evolves $q(x)$ using the Altarelli-Parisi equations and performs a global fit. The region below 10^{-2} had not been explored experimentally until very recently; a first look at these low- x values has been provided by the commissioning of HERA.

HERA is a machine ideally suited for an in-depth analysis of structure functions. Fig. 34 shows the physics reach of HERA. It should be possible to arrive at very low values of x (up to $x \sim 10^{-4}$). There are already very interesting results on the region $10^{-3} < x < 10^{-2}$, which actually show that most of the parametrizations of the structure functions perform very poorly when it comes to reproducing the data at low- x . Typically they predict an increase as $x \rightarrow 0$ which is lower than what is actually seen (fig. 35). The behaviour $F_2(x) \sim x^{-\lambda}$, with $\lambda \sim 1/2$ as $x \rightarrow 0$, which is predicted from the Kuraev-Fadin-Lipatov evolution equation³⁵⁾ seems to stand the comparison with HERA results

best. However, this behaviour is still incompatible with unitarity and cannot hold all the way to $x = 0$ either. Obviously there are many open questions in this area.

Fig. 34.- The physics reach of HERA. From³⁷⁾.

Fig. 35.- Low x structure functions at HERA³⁸⁾.

Complementary information on the parton distribution functions is provided by the use of sum rules. If one of the operators that appear in the OPE corresponds to a conserved current such as, for instance, the non-singlet operator

$$\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d. \quad (145)$$

The corresponding anomalous dimension vanishes and its expectation value corresponds to a conserved charge measured in the nucleon state³⁹⁾. Then we know

$$\int dx x^{-1} F^{(NS)}(x) = A_1 c_1 \quad (146)$$

for non-singlet operators and a linear combination of

$$\int dx F^{(S)}(x) \quad (147)$$

for the singlet operators. In this case the conserved current is the energy-momentum tensor. As an example let us consider ep scattering. The non-singlet conserved current is $\bar{\psi}\gamma_\mu Q\psi$ (Q being the charge matrix) and the associated conserved charge is the nucleon electric charge. Then

$$\int dx x^{-1} F_2^{ep}(x) = \frac{1}{3}(1 + \mathcal{O}(\alpha_s)). \quad (148)$$

Other sum rules are

$$\int dx x^{-1} (F_2^{\bar{\nu}p}(x) - F_2^{\nu p}(x)) = 0, \quad (149)$$

due to Adler, or the Gross-Llewellyn-Smith sum rule (I : isoscalar target)

$$\int dx x^{-1} F_3^{\nu I}(x) = 3(1 + \mathcal{O}(\alpha_s)). \quad (150)$$

16.- Large Rapidity Gaps

It would not be appropriate to conclude this review without saying a few words about some very interesting events that have been seen at HERA⁴⁰⁾.

We discussed in section 7 the phenomenon of color coherence. In DIS we expect that hadron production should take place predominantly in the color string joining the struck parton and the nucleon remains. In practice this means that most hadron energy deposition should occur in an angular region relatively close to the proton beam direction. Described in terms of the pseudorapidity $\eta = -\log \tan \theta/2$ (θ being the polar angle measured following the incoming proton) one should expect hadron energy deposition to be at relatively large values of η ($\eta_{max} = 4.3$).

While this is the case for the bulk of the event sample (fig. 36), a small but sizeable fraction remains all the way to $\eta \sim -2$, with little hadronic activity in between. These ‘large rapidity gap’ events *cannot* be accounted for in the parton model we have discussed.

It has been suggested that the large rapidity gap events can be explained in terms of the pomeron⁴¹⁾, which is believed to dominate elastic and diffractive production in hadron-hadron interactions. Although the pomeron itself falls completely outside perturbative QCD (presumably you have to sum infinite families of diagrams to reproduce pomeron-like behaviour), the concept of pomeron structure functions has been put forward and studied. The idea is that the pomeron could exhibit a partonic structure that could be probed in hard diffractive processes. The events observed at HERA appear to be of this kind.

Fig. 36.- Rapidity distribution of events

17.- Summary and Outlook

QCD is now twenty years old. Obviously we are a long way from the days when it was hotly debated whether a sensible Field Theory could possibly be asymptotically free. We know today that QCD makes perfect sense, in fact QCD is now the paradigm of a Quantum Field Theory and the primary suspect is QED, which is believed not to be a consistent theory all the way to zero distances. This is indeed a paradoxical situation.

Quantum Chromodynamics is a strongly coupled theory. Ultimately this can be traced to the fact that Λ_{QCD} is similar in magnitude to the light quark masses. For very heavy quarks perturbation theory gives reasonable estimates, but fails for light quarks where confinement plays a crucial role.

Then it would have seemed rather hopeless twenty years ago to expect that we would be able to measure α_s with a mere 5% error. Confinement makes most hadronic observables to depend on α_s in an unknown form. Non-perturbative studies such as lattice QCD can, in principle, unveil the α_s dependence, but we are still a long way from reaching the above precision, even though these studies provide overwhelming evidence that QCD does describe the hadronic world.

Although many valuable theoretical developments exist by now, it is through experiment that we have ultimately learnt most of what we know about QCD and α_s . At LEP we are in the privileged situation of having an experiment with high statistics, a clean set-up and typical processes with a large momentum transfer, leading to a reasonable convergence of perturbative expansions for inclusive processes. Thanks to this triple conjunction we have been able to reach the level of accuracy of a few per cent.

Yet, with practically everything computed at the next-to-leading order and with no other experiment of similar quality in sight, it is hard to imagine a sizeable reduction in the error of α_s in the near future.

HERA will probably capture most of the interest of workers in the field in years to come. There we have the possibility of exploring a new kinematic range in Deep Inelastic Scattering. Sensitivity to the low- x range in structure functions will allow us to explore the region where parton wave functions begin to overlap and perturbation theory fails to describe the physics even when $Q^2 \rightarrow \infty$.

A spectacular evidence for the insufficiency of perturbation theory even for processes with large Q^2 is to be seen in the detection of large rapidity gap events, interpreted in terms of collective modes of dual theory. This is not really new; Regge theory has been

long required to understand the elastic and low p_T behaviour of hadron interactions. What is new and interesting is that at HERA we will probably be able to interpolate smoothly between the Regge limit of QCD and ordinary perturbation theory. Many surprises are probably in store for us.

Acknowledgements

These lectures were written up during the author's visit to the Theory Group of Fermi National Laboratory whose hospitality is gratefully acknowledged. We thank J.Fuster and Ll.Garrido for several discussions concerning the determination of α_s . Thanks are also due to R.Tarrach for reading the manuscript, although the author is solely responsible of any remaining mistakes. The financial support of CICYT grant AEN93-0695 and CEE grant CHRX CT93 0343 is gratefully acknowledged.

Bibliography

- 1) See for instance: F.J.Ynduráin, *The Theory of Quark and Gluon Interactions*, Springer Verlag; G.Sterman, *Quantum Field Theory*, Cambridge University Press, T. Muta, *Foundations of Quantum Chromodynamics*, World Scientific.
- 2) J.Fuster, these Proceedings.
- 3) D.Adams et al. (the SMC collaboration), CERN-PPE-94-59; For theoretical discussions see for instance: G. Altarelli, in Proceedings of the International School of Subnuclear Physics, Erice 1989; A. Manohar, in Proceedings of the Polarized Collider Workshop, University Park, 1990.
- 4) See for instance: E.Laenen et al, Phys. Rev.D49 (1994) 5753, and references therein.
- 5) M.Aguilar-Benitez et al. (the Particle Data Group), in Review of Particle Properties, Phys. Rev. D45 (1992) S1.
- 6) In addition to the references listed in ¹⁾ see also: J.D.Bjorken and S.Drell, *Relativistic Quantum Fields*, Mc.Graw-Hill; C.Itzykson and J.B. Zuber, *Quantum Field Theory*, Mc-Graw-Hill.
- 7) See for instance: R. Jackiw in *Current Algebra and Anomalies*, Princeton.
- 8) For a simple exposition see: S.Coleman, in *Aspects of Symmetry*, Cambridge University Press.
- 9) For a more detailed discussion and further references see: E. de Rafael, in Proceedings of the GIFT Workshop on Quantum Chromodynamics, Jaca, 1979, Springer Verlag.
- 10) M.Gell-Mann and Y.Ne'eman, in *The Eightfold Way*, Benjamin.
- 11) See e.g.: S.Coleman, in ⁸⁾
- 12) T.Hebbeker, Physics Reports 217 (1992) 69
- 13) Although in the context of weak interactions a clear discussion is given in: J.C.Taylor, in *Gauge Theories of Weak Interactions*, Cambridge University Press
- 14) C.Callan, Phys. Rev. D2 (1970) 1541; K.Symanzik, Com. Math. Phys. 18 (1970) 227; see also ⁹⁾
- 15) For a discussion and further references: F.J.Ynduráin in ¹⁾; see also S.Weinberg, Phys. Rev. D8 (1973) 3497.
- 16) F.Wilczek, in Proceedings of the Lepton-Photon Conference, Ithaca, 1993.

- 17) For a recent comprehensive work see: S.Titard and F.J.Ynduráin, Phys. Rev D49 (1994) 6007 and FTUAM-94-6.
- 18) H. Georgi, in *Weak Interactions and Modern Particle Theory*, Benjamin-Cummings.
- 19) H.Leutwyler, Phys. Lett. 98B (1981) 447.
- 20) See e.g.: R.K.Ellis and W.J.Stirling, Lectures given at the CERN School of Physics, FERMILAB-Conf-90/164-T.
- 21) J.Donoghue, E.Golowich and B. Holstein, in *Dynamics of The Standard Model*, Cambridge University Press.
- 22) See e.g.: A.C. Mattingly and P.M.Stevenson, Phys. Rev. D49 (1994) 437.
- 23) E. Braaten, S.Narison and A.Pich, Nucl. Phys. B373 (1992) 581.
- 24) F.Bloch and A. Nordsieck, Phys. Rev. 52 (1937) 54.
- 25) T.Kinoshita, J. Math. Phys. 3 (1962) 650; T.D.Lee and M.Nauenberg, Phys. Rev 133B (1964) 1549.
- 26) T.Appelquist et al. Phys. Rev. Lett. 36 (1976) 768; Nucl. Phys. B120 (1977) 77. See also G.Sterman in ¹⁾.
- 27) N.Magnoli, P.Nason and R.Rattazzi, Phys. Lett. B252 (1990) 271.
- 28) G.Miller et al. (the SLAC-MIT collaboration), Phys. Rev. D5 (1972) 528.
- 29) See e.g. ref ²⁰⁾ or F.J.Ynduráin in ¹⁾.
- 30) M.Derrick et al. (the ZEUS collaboration), Phys. Lett. B315 (1993) 481.
- 31) Our discussion follows T. Muta in ref ¹⁾.
- 32) K.Wilson, Phys. Rev. 179 (1969) 1499. For a detailed practical example see: P.Pascual and R.Tarrach in *QCD: Renormalization for the Practitioner*, Springer Verlag.
- 33) A.C.Benvenuti et al., Phys. Lett. B223 (1989)485.
- 34) G.Altarelli and G.Parisi, Nucl. Phys. B126 (1977) 298.
- 35) E.Kuraev, L.Lipatov and V.Fadin, Sov. Phys. JETP 45 (1977) 199; see also E.Levin, in *Proceedings of the Blois Conference on Elastic and Diffractive Scattering*, Providence, 1993
- 36) A.Martin, W.J.Stirling and R.Roberts, Phys. Rev. D 47 (1993) 867
- 37) G.A.Schuler, in *Physics at HERA*, Vol. 1
- 38) T. Ahmed et al (the H1 collaboration), Nucl. Phys. B407 (1993) 515; Phys. Lett. B321 (1994) 161
- 39) See ¹⁾. See also: A.J.Buras, Rev. Mod. Phys. 52 (1980) 199
- 40) L.Labarga, these Proceedings; M.Derrick et al. (the ZEUS collaboration), Phys. Lett. B332 (1994) 228; T.Ahmed et al. (the H1 collaboration), DESY-94-133
- 41) See e.g. A.Donnachie and P.V.Landshoff, Nucl. Phys. 231 (1984) 189; Nucl. Phys. B267 (1986) 690
- 42) G.Ingelman and P.Schlein, Phys. Lett. B152 (1985) 256; E. Berger et al., Nucl. Phys. B 286 (1987) 704